Journal of Engineering Physics and Thermophysics, Vol. 78, No. 2, 2005

PROPAGATION OF HEAVY ATMOSPHERIC EMISSIONS WITH ALLOWANCE FOR THE GROUND LANDSCAPE

V. Sh. Shagapov, S. M. Mukhametshin, and G. R. Galiaskarova

UDC 539.532

A study of the influence of flora on the process of propagation of heavy atmospheric emissions with allowance for the ground landscape has been made.

Introduction. Creation of mathematical models of propagation of heavy atmospheric emissions (whose average density is higher than the atmospheric-air density) near the earth's surface has been pressing in the last few years, particularly for cities with a developed chemical industry. The level of contamination of such cities depends not only on the number of emissions but on weather conditions as well. Among the main meteorological factors affecting the distribution and transfer of harmful substances that have arrived at the atmospheric bottom layer are [1]: the direction and force of a wind, the temperature and radiation regimes, and atmospheric phenomena and precipitation. The wind force largely determines the removal of air contaminants beyond the limits of a city. Weak winds and still conditions contribute to the accumulation of harmful substances in the atmospheric bottom layer. The interaction of a moving air mass and the earth's surface may result in so-called "poorly ventilated" zones in which accumulation of the impurity and quite a significant concentration growth with further transfer of the impurity to other portions of the area are possible [2].

The distinctive features of propagation of mixtures consisting of solid particles or liquid aerosols (particulate mixtures) are also determined by the presence of the intrinsic settling rate in them, which can be determined from the Stokes formula:

$$\upsilon_{\mathrm{p}} = \frac{2\rho_{\mathrm{p}} g}{9\mu} a_{\mathrm{p}}^2 ,$$

whence, in particular, it follows that for a dust cloud with $\rho_{\mathrm{p}} = 2.3 \cdot 10^3$ kg/m^3 (silicon density) and at $a_{\mathrm{p}} = 1$, 10, and 50 μm we obtain $\upsilon_{\mathrm{p}} = 3 \cdot 10^{-4}$, $3 \cdot 10^{-2}$, and 0.7 m/sec respectively.

Solid particles formed at negative temperatures because of the freezing of water-containing droplets very often represent dendrite systems with a "poor" aerodynamics (with large coefficients of resistance). As a consequence, the sedimentation rates of such particles will be low, although their effective radii reduced to spherical particles can be larger than tens of microns.

In this work, we will disregard the settling of particles due to their intrinsic sedimentation rate relative to air, thus assuming that the corresponding two-phase systems are fairly finely divided or that suspended particles with a "poor" aerodynamics have large coefficients of resistance. Therefore, we will consider mixtures of gases or a gas suspension with different aerosol particles as a certain reduced gas with a higher density than that of the ambient atmospheric air. As a consequence, such mixtures will propagate along the earth's surface.

Mathematical Model. The process of propagation of atmospheric emissions will be considered based on quasi-two-dimensional equations obtained analogously to the "shallow"-water theory [3, 4].

We locate the coordinate plane xOy perpendicularly to the direction of gravity and guide the Oz axis vertically upward. The average emission density will be denoted by ρ, whereas the atmospheric-air density will be denoted by ρ_{a}. Then the mass and momentum equations for the emission layer of height h counted off from the earth's surface will be written in the form

Sterlitamak State Pedagogical Institute, 49 Lenin Ave., Sterlitamak, 453103, Bashkortostan, Russia; email: shagapov@rambler.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 2, pp. 99–103, March–April, 2005. Original article submitted November 28, 2003.

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = 0 , \tag{1}$$

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot (\upsilon \otimes \mathbf{q}) = - g' h \nabla H - \mathbf{f} , \tag{2}$$

$$H (x, y, t) = z_0 (x, y) + h (x, y, t) , \quad g' = \frac{\rho - \rho_a}{\rho} g , \quad \mathbf{q} = h\upsilon , \quad \nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} ,$$

$$\nabla \cdot (\upsilon \otimes \mathbf{q}) = \left(\frac{\partial}{\partial x} (\upsilon_x q_x) + \frac{\partial}{\partial y} (\upsilon_y q_x) , \ \ \frac{\partial}{\partial x} (\upsilon_x q_y) + \frac{\partial}{\partial y} (\upsilon_y q_y) \right) .$$

Here $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the coordinate axes Ox and Oy respectively.

Equations (1) and (2) have been written under the assumption that there is no mass exchange between the discharge volume and the ambient atmospheric air.

Assignment of Friction Forces. Among the factors affecting the process of spreading is flora. Then we will recognize two situations in assigning friction forces. In the case of propagation of emissions, for example, in a forest, when the height of trees is larger than the height of the plume, the force of resistance from the trees will be used in the form of the square law [5]

$$\mathbf{f} = \frac{\upsilon \, |\upsilon| \, h}{r_\upsilon} , \tag{3}$$

where r_υ is the empirical parameter. In particular, if we take the trees as cylindrical bodies, then, using the formulas of flow across cylinders for large Reynolds numbers, we may assume that $r_\upsilon = \chi(nd)$.

When the height of the plume of atmospheric emissions is much larger than the height of ground-based objects or when these objects are absent, the resistance force will be determined as follows:

$$\mathbf{f} = \lambda \upsilon \, |\upsilon| \, . \tag{4}$$

There are two approaches in assigning the coefficient λ. According to the first approach, the coefficient of resistance λ will be set constant. In the second case it will be determined from the Manning law [6]:

$$\lambda = \left(\frac{h_*}{h} \right)^{1/3} , \tag{5}$$

where h_* is the parameter responsible for the roughness of the surface.

Numerical Algorithms. System (1), (2) has been solved by the finite-difference method on a fixed uniform rectangular grid. To approximate the equations we used a difference scheme that is an analog of the "cross" scheme well known in gas dynamics [7, 8]. It is noteworthy that the initial system of equations allows discontinuous solutions. To calculate such solutions without separating explicitly the discontinuity front on the grid one uses the method of "smearing" of the front by introduction of the so-called pseudoviscosity or artificial viscosity into the system of differential equations [9]. A continuous variation in all the parameters on the shock-wave front is produced by the action of such a viscosity. The artificial viscosity $\omega(\omega_x, \omega_y)$ is introduced into the right-hand side of the momentum equation (2) analogously to [8]; we have

$$\omega_x = C_1 \left[\frac{\partial}{\partial x} \left(|q_x| \, \frac{\partial \upsilon_x}{\partial x} \right) + \frac{1}{2} \frac{\partial}{\partial y} \left(|q_x| \, \frac{\partial \upsilon_x}{\partial y} \right) \right] ,$$

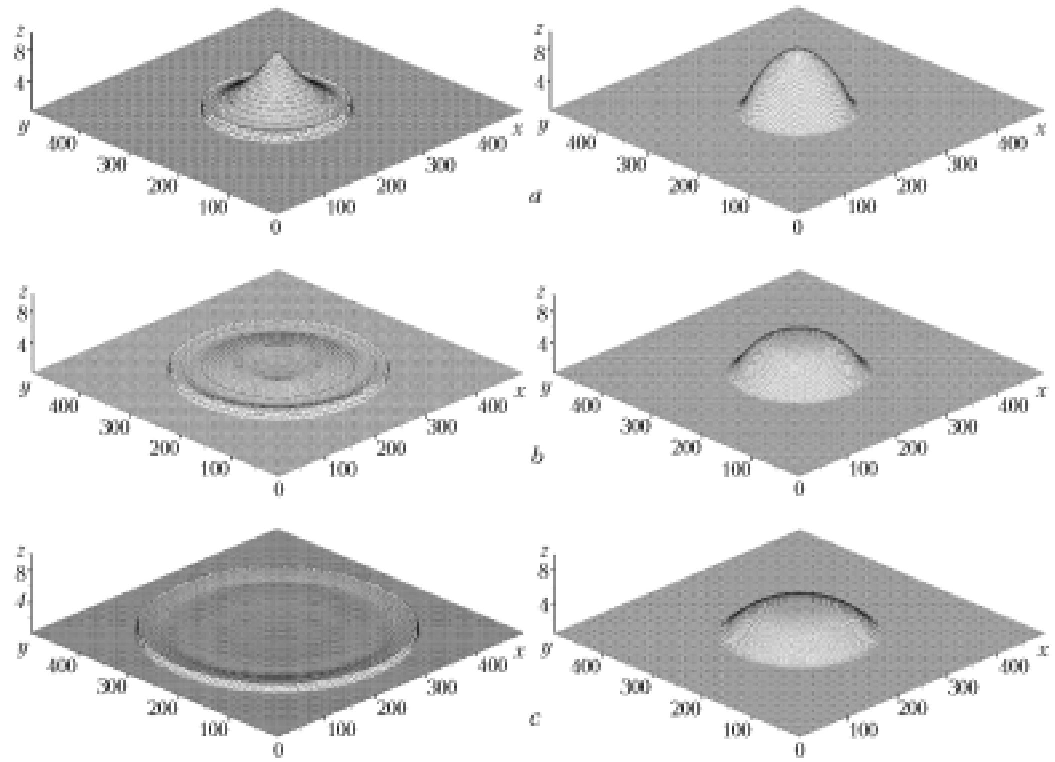

Fig. 1. Shapes of the free emission surface in the absence of flora (on the left) and with allowance for the influence of flora (on the right) at different instants of time: a) t = 1; b) 2; c) 3 min.

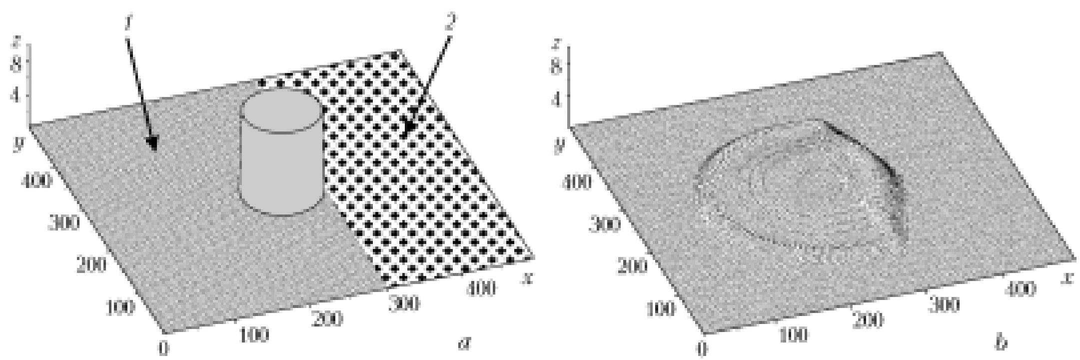

Fig. 2. Case of a nonuniform distribution of flora on the earth's surface.

$$\omega_y = C_2 \left[\frac{\partial}{\partial y} \left(|q_y| \frac{\partial \upsilon_y}{\partial y} \right) + \frac{1}{2} \frac{\partial}{\partial x} \left(|q_y| \frac{\partial \upsilon_y}{\partial x} \right) \right],$$

where C_1 and C_2 = const. With the optimum selection of the coefficients of artificial viscosity C_1 and C_2, the difference scheme smears the discontinuities into three to four space intervals without pronounced numerical oscillations.

Calculation Results. We give results of calculations of the spreading of heavy atmospheric emissions along the earth's underlying surface (underlying terrain). We assume that, at the initial instant of time, the emission volume is in a reservoir of height h_0, standing on the earth's surface. Then, after the instantaneous opening of the walls, it begins to spread by the action of gravitational forces. In the calculations, we assumed that the density of the atmospheric air is ρ_a = 1.3 kg/m^3 and the average density of the emission is ρ = 1.4 kg/m^3. We used the following parameters in assigning the resistance forces: for the empirical parameter involved in the resistance law (3), we took the value r_υ = 5 m, which corresponds to a forest density of n = 0.4 m^{-2} (average distance between trees 3 m) with a

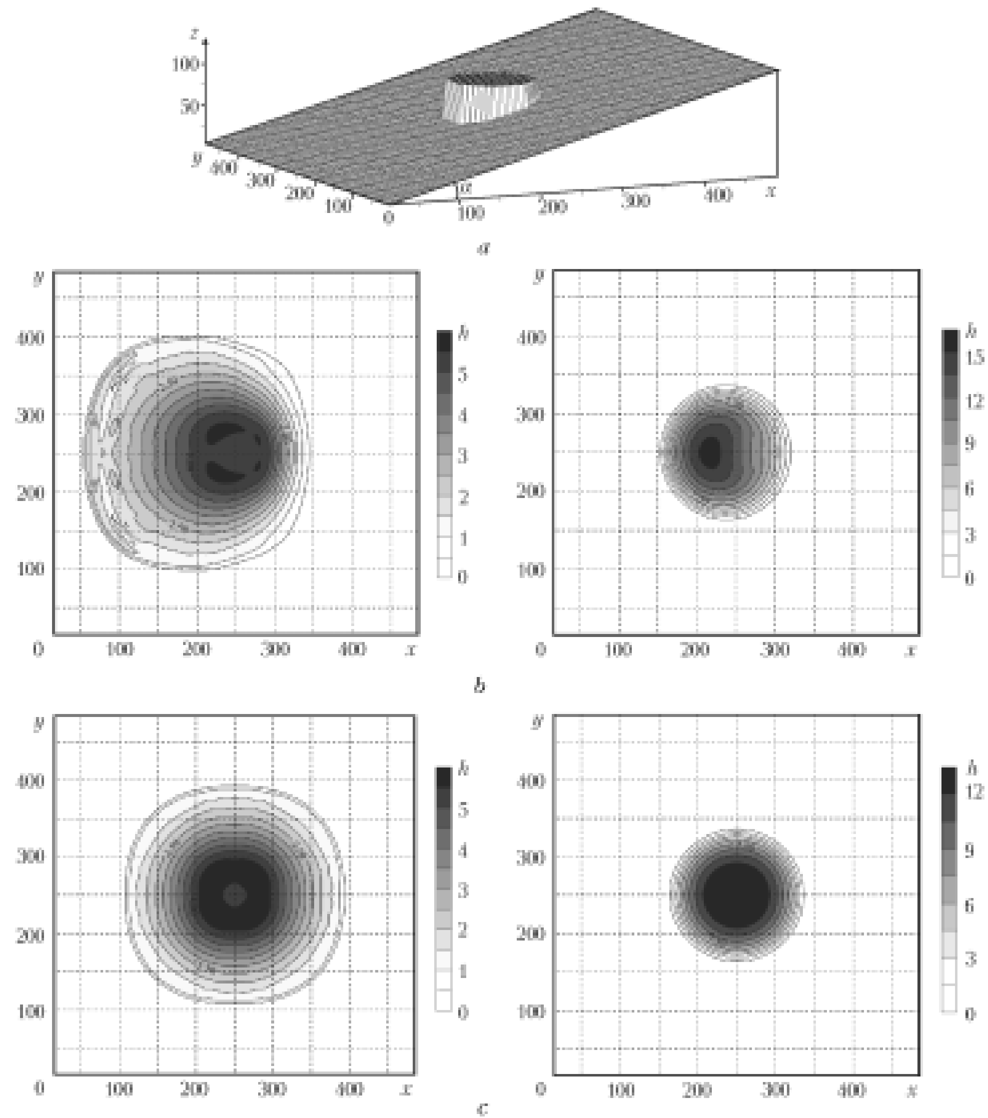

Fig. 3. Pattern of spreading over the sloping surface of the earth: a) initial position of the emission; b) $\alpha = 15^o$; c) 0^o.

characteristic diameter of the trees $d = 0.5$ m. For the resistance law (4), based on the data given in [10] we take the value $\lambda = 2.5 \cdot 10^{-3}$ for the parameter λ, which corresponds to a flat field, whereas the value $h_* = 2.3 \cdot 10^{-7}$ m is taken for the parameter h_* in the Manning law (5).

The calculations show that, in propagation of emissions without allowance for the influence of flora, both approaches give virtually identical results in assigning the coefficient λ (the relative difference in the coordinates of the plume boundary amounts to no more than ten percent according to these two schemes). Therefore, in further examples, the calculations presented have been performed at $\lambda = \text{const}$ (first scheme).

Figure 1 illustrates the influence of flora on the dynamics of spreading over the horizontal surface of the earth. At the initial instant of time, the emission is in a cylindrical reservoir of height $h_0 = 10$ m and base radius $R = 50$ m. It is seen that the presence of flora strongly retards the process of spreading: the radii of the boundaries of zones covered with the plume in the presence of flora have decreased approximately three times for the same instants of time. The plume height is accordingly several times larger.

Using this mathematical model, we can also calculate the dynamics of propagation of heavy emissions in the case where the distribution of flora over the earth's surface is nonuniform. Figure 2 shows the case of spreading of emission in the presence of a forest belt: Fig. 2a is the situation at the initial instant of time and Fig. 2b is that at the

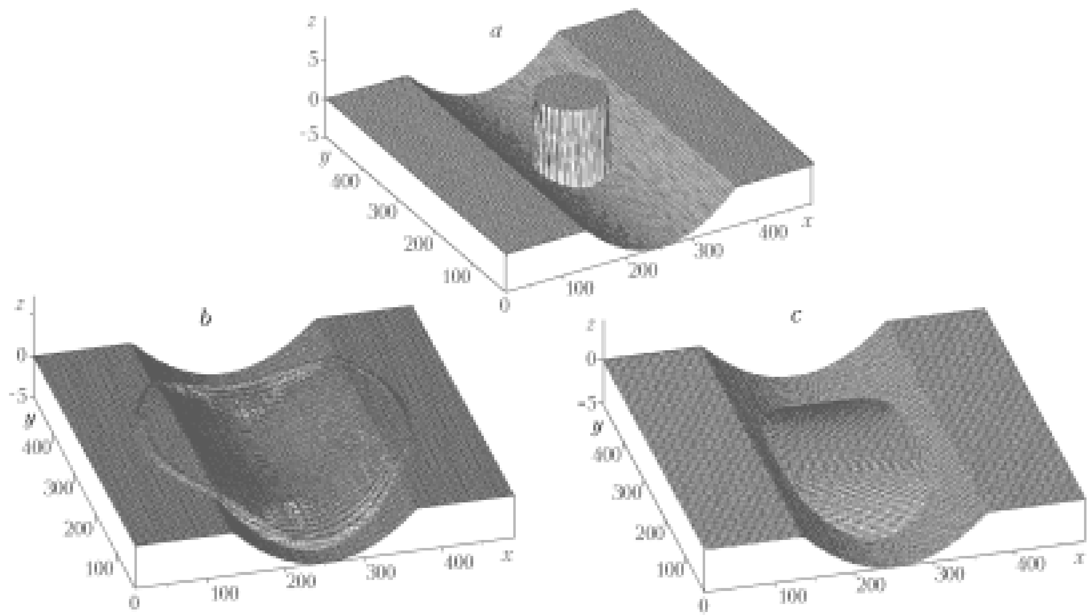

Fig. 4. Pattern of spreading of the emission over the inhomogeneous surface of the earth: a) at the initial instant of time; b) without allowance for the influence of flora at t = 2 min; c) with allowance for the presence of flora at t = 3 min.

instant of time t = 1 min. The earth's surface is a horizontal plane; region 1 corresponds to the case where flora is absent; in region 2, flora is assumed to be present. In the forest belt, the height turned out to be approximately 2.5 times larger.

Figure 3 gives the emission distribution for the case where the underlying surface is a plane located at an angle α to the horizon. Figure 3a shows the initial position of the emission volume: it is a truncated right cylinder; the upper base is horizontal, whereas the lower base is on the earth's surface. This cylinder's height (we take the distance between the centers of the cylinder bases as the height) is h_0 = 25 m; the radius of the upper base is R = 50 m. Figure 3b and c gives the level lines of the characteristic height h of the emission on the earth's surface; the figures on the left correspond to the case where flora is absent, and those on the right correspond to the presence of flora. All the figures are presented for one instant of time: t = 30 sec. In this example, the slope of the earth's surface virtually does not affect propagation in the presence of flora. The calculations show that the characteristic height of the emission increases with slope, whereas the position of the center of gravity of the plume shifts along the slope.

Figure 4 illustrates the case where the volume of emission having the shape of a cylinder of height h_0 = 10 m and radius R = 50 m is in a "ravine" (Fig. 4a) of width 200 m and height 5 m (half the initial height h_0) at the initial instant of time. Figure 4b and c gives the patterns of distribution of the characteristic height h of the emission over the earth's surface for different resistance laws. It is seen that the emission volume cannot leave the "ravine" in the presence of flora — it settles in it. If flora is absent, the presence of the "ravine" virtually has no effect on the propagation of emissions.

Conclusions. Based on the quasi-two-dimensional "shallow water" equations, we have revealed certain qualitative and quantitative features of the propagation of heavy (as compared to the ambient air) atmospheric emissions under the action of gravitational forces. It has been shown that the presence of flora strongly retards the propagation of such emissions and to localize them one can use, first, the distinctive features of natural and artificially produced profiles of the ground, and, second, forest stands of a definite configuration.

NOTATION

a_p, particle radius, μm; C_1 and C_2, coefficients of artificial viscosity, m; d, characteristic diameter of trees, m; $\mathbf{f}(f_x, f_y)$, force of resistance from the earth's surface and ground-based objects, m^2/sec^2; g, free-fall acceleration, m/sec^2;

$H(x, y, t)$, coordinate of the upper boundary of the emission volume, m; h_*, parameter responsible for the roughness of the earth's surface, m; h_0, initial height, m; n, number of trees per unit area (forest density), 1/m^2; v_υ, empirical parameter, m; $\mathbf{q}(q_x, q_y)$, flow rate, m^2/sec; R, radius, m; t, time, sec; x, y, z, Cartesian coordinates, m; $z_0(x, y)$, function assigning the shape of the earth's surface, m; α, angle of slope, deg; ρ_p, particle density, kg/m^3; μ, coefficient of dynamic viscosity of air, Pa·sec; ρ, average emission density, kg/m^3; ρ_a, atmospheric-air density, kg/m^3; $\upsilon(\upsilon_x, \upsilon_y)$, emission rate, m/sec; υ_p, rate of settling of finely divided particles, m/sec; χ, dimensionless coefficient ($\chi \sim 1$); λ, coefficient of resistance from the earth's surface; $\omega(\omega_x, \omega_y)$, artificial viscosity, m^2/sec^2. Subscripts: p, particle; a, atmospheric.

REFERENCES

1. T. R. Oke, *Boundary Layer Climates* [Russian translation], Gidrometeoizdat, Leningrad (1982).
2. M. G. Korotkov, Modeling of scenarios of daily variation in atmospheric circulation near a town and in an industrial region, *Vych. Tekhnol.*, **7**, 57–63 (2002).
3. J. J. Stoker, *Water Waves* [Russian translation], Moscow–Leningrad (1959).
4. R. E. Britter, The spread of a negatively buoyant plume in a calm environment, *Atmosph. Environ.*, **13**, 1241–1247 (1979).
5. É. L. Benovitskii, Hydraulic drag at the boundary of the thicket of higher vegetation in open river beds, *Vodn. Resur.*, No. 3, 71–75 (1991).
6. R. R. Chugaev, *Hydraulics* [in Russian], Énergiya, Leningrad (1975).
7. O. M. Belotserkovskii and Yu. M. Davydov, *Method of Coarse Particles in Gas Dynamics. Computational Experiment* [in Russian], Nauka, Moscow (1982).
8. V. V. Ostapenko, Numerical simulation of wave flows due to the trailing of a shore landslide, *Zh. Prikl. Mekh. Tekh. Fiz.*, **40**, No. 4, 109–117 (1999).
9. A. A. Samarskii and Yu. P. Popov, *Difference Methods for Solving Problems of Gas Dynamics* [in Russian], Nauka, Moscow (1980).
10. V. Sh. Shagapov and G. R. Geliaskarova, On the dynamics of accumulation of atmospheric emissions of a negative buoyancy in windless weather, *Inzh.-Fiz. Zh.*, **75**, No. 2, 22–27 (2002).